

## PORTFOLIO ANALYSIS

A portfolio can be viewed as a combination of assets held by an investor.

For each asset held, such as company stocks, the logarithmic or continuously compounded **rate of return**  $r$  at time  $t$  is given by

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$

where  $P_t$  is the stock price at time  $t$ , and  $P_{t-1}$  is the stock price in the prior period.

The **volatility of stock returns**, over period  $N$  is often estimated by the sample variance  $\sigma^2$

$$\sigma^2 = \sum_{t=1}^N \frac{(r_t - \bar{r})^2}{n-1}$$

where  $r_t$  is the return realized in period  $t$ , and  $N$  is the number of time intervals. As the variance of returns is in units of percent squared, we take the square root to determine the standard deviation  $\sigma$ .

### Example (file: xlf-portfolio-analysis-v2.xlsm)

Suppose an investor has a four stock portfolio comprising of shares on the Australian stock market as listed in Figure 1.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Portfolio of four Australian listed companies												
6													
7	Stock prices \$ (adjusted - closing), frequency: monthly												
8	Date	AGK	CSL	SPN	SKT		AGK	CSL	SPN	SKT			
9	2/04/2012	14.98	36.66	1.11	4.27		1.55%	2.12%	2.74%	3.82%		J9: =LN(E9/E10)	
10	1/03/2012	14.75	35.89	1.08	4.11		5.58%	9.06%	7.70%	4.73%			
11	1/02/2012	13.95	32.78	1.00	3.92		-4.49%	5.16%	4.08%	-3.02%			
12	3/01/2012	14.59	31.13	0.96	4.04		1.80%	-2.76%	2.11%	1.00%			
13	1/12/2011	14.33	32.00	0.94	4.00		0.49%	2.56%	0.00%	-2.23%			
14	1/11/2011	14.26	31.19	0.94	4.09		-0.98%	7.76%	-6.19%	-2.18%			
15	3/10/2011	14.40	28.86	1.00	4.18		0.70%	-2.80%	6.19%	-1.66%			
16	1/09/2011	14.30	29.68	0.94	4.25		-7.93%	5.40%	1.07%	-7.26%			
17	1/08/2011	15.48	28.12	0.93	4.57		8.70%	-8.75%	1.08%	-0.44%			
18	1/07/2011	14.19	30.69	0.92	4.59		-3.19%	-7.44%	-2.15%	6.99%			
19	1/06/2011	14.65	33.06	0.94	4.28		1.93%	-2.51%	1.07%	-1.62%			
20	2/05/2011	14.37	33.90	0.93	4.35		-1.18%	-1.32%	5.53%	2.33%			
21	1/04/2011	14.54	34.35	0.88	4.25								
22							AGK	CSL	SPN	SKT			
23						Minimum	-0.07929	-0.08746	-0.06188	-0.07259		J23: =MIN(J9:J20)	
24						Maximum	0.087011	0.09064	0.076961	0.069927		J24: =MAX(J9:J20)	
25						Average	0.002484	0.005424	0.019349	0.000391		J25: =AVERAGE(J9:J20)	
26						Std Dev.p	0.041997	0.054857	0.036297	0.037564		J26: =STDEV(J9:J20)	
27						Std Dev.s	0.043864	0.057296	0.037911	0.039234		J27: =STDEV(J9:J20)	
28						Variance.p	0.001764	0.003009	0.001317	0.001411		J28: =VAR(J9:J20)	
29						Variance.s	0.001924	0.003283	0.001437	0.001539		J29: =VAR(J9:J20)	

Fig 1: Excel functions - descriptive statistics

The stock codes AGK, CSL, SPN, and SKT from figure 1 are described in figure 2.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Portfolio of four Australian listed companies													
2		AGK	AGL Energy Ltd: operates Australia's largest retail energy and dual fuel customer base											
3		CSL	CSL Limited: pharmaceutical and diagnostic products											
4		SPN	SP Ausnet: Electricity transmission and distribution and gas distribution											
5		SKT	SKY Network Television Limited: New Zealand's pre-eminent pay television operator.											
6														

Fig 2: Portfolio components - description

## 1. Descriptive statistics

Price data for the four stocks is obtained from Yahoo Finance and is filtered for monthly price observations. In figure 1, Column A of the worksheet shows the date for the first trading day of the month. Closing prices for the four stocks are in the range B9:E21. The corresponding continuously compounded return series, using the Excel LN function, are calculated in the range G9:J21. Summary information from Excel statistical functions are shown in rows 23 to 27, using the Excel 2007 formulas for standard deviation and variance (the Excel 2010 equivalent formula is in column F).

Descriptive statistics can also be produced by using the **Descriptive Statistics** item from the Data Analysis dialog as shown in figure 3.

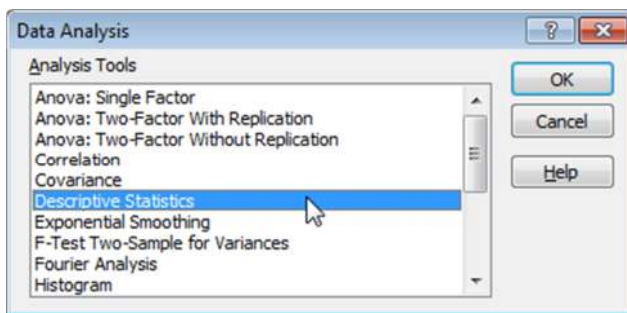


Figure 3: Data Analysis dialog box – with Descriptive Statistics selected

The output for the Descriptive Statistics is shown in the New Worksheet ply in figure 4.

	A	B	C	D	E	F	G	H	I
1	AGK		CSL		SPN		SKT		
2									
3	Mean	0.002484	Mean	0.005424	Mean	0.019349	Mean	0.000391	
4	Standard Error	0.012662	Standard Error	0.016540	Standard Error	0.010944	Standard Error	0.011326	
5	Median	0.005933	Median	0.004020	Median	0.015932	Median	-0.010295	
6	Mode	#N/A	Mode	#N/A	Mode	0.010695	Mode	#N/A	
7	Standard Deviation	0.043864	Standard Deviation	0.057296	Standard Deviation	0.037911	Standard Deviation	0.039234	
8	Sample Variance	0.001924	Sample Variance	0.003283	Sample Variance	0.001437	Sample Variance	0.001539	
9	Kurtosis	0.668948	Kurtosis	-0.981764	Kurtosis	0.744701	Kurtosis	-0.012231	
10	Skewness	0.087421	Skewness	-0.114118	Skewness	-0.577340	Skewness	0.103161	
11	Range	0.166301	Range	0.178096	Range	0.138836	Range	0.142521	
12	Minimum	-0.079289	Minimum	-0.087456	Minimum	-0.061875	Minimum	-0.072594	
13	Maximum	0.087011	Maximum	0.090640	Maximum	0.076961	Maximum	0.069927	
14	Sum	0.029813	Sum	0.065084	Sum	0.232193	Sum	0.004695	
15	Count	12	Count	12	Count	12	Count	12	
16									

Figure 4: Analysis ToolPak (data analysis) - descriptive statistics

Rows 7 and 8 of the worksheet shown in figure 4 have values for the sample standard deviation and sample variance respectively (rows 7 and 8). We will see later, that the **Data Analysis > Covariance** item returns population values, not sample values.

When assets are held as part of a portfolio, an important consideration is the amount of co-movement between portfolio components.

## 2. Covariance

The amount of co-movement can be measured by the covariance statistic, and is calculated on a pairwise basis. The formula for the sample covariance  $\sigma_{i,j}$  for the return vectors of stock  $i$  and stock  $j$  is

$$\sigma_{i,j} = \sum_{t=1}^N \frac{(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j)}{n - 1}$$

There are number of ways the estimation can be operationalized and some techniques are described in this section. Methods include the Analysis ToolPak – Covariance item (figure 3), and Excel functions listed here.

### Excel 2007

COVAR(array1,array2)	Returns covariance, the average of the products of paired deviations
----------------------	--

### Excel 2010

COVARIANCE.S(array1,array2)	Returns the sample covariance, the average of the products deviations for each data point pair in two data sets
COVARIANCE.P(array1,array2)	Returns covariance, the average of the products of paired deviations

The worksheet in figure 7 shows output for the Analysis ToolPak (ATP) covariance item in rows 32 to 36. The covariance matrix, from the ATP is a lower triangular table, meaning it only returns the main diagonal elements, and the lower left elements. By definition, the covariance of a vector with itself, is the variance of the vector. Thus, the value in cell G33 in figure 5,  $\sigma_{AGK,AGK} = 0.001764$ , is the same value as the population variance returned by the Excel VARP function shown in figure 1 cell G28.

	A	B	C	D	E	F	G	H	I	J	K	L	M
31													
32							AGK	CSL	SPN	SKT			
33						AGK	0.001764						
34						CSL	-0.00069	0.003009					
35						SPN	0.000338	0.000127	0.001317				
36						SKT	0.000608	-0.0006	0.000223	0.001411			
37													
38													
39							AGK	CSL	SPN	SKT			
40						AGK	0.001764	-0.000689	0.000338	0.000608		J40: =COVAR(AGK,SKT)	
41						CSL	-0.000689	0.003009	0.000127	-0.000599		J41: =COVAR(CSL,SKT)	
42						SPN	0.000338	0.000127	0.001317	0.000223		J42: =COVAR(SPN,SKT)	
43						SKT	0.000608	-0.000599	0.000223	0.001411		J43: =COVAR(SKT,SKT)	
44													

**Fig 5: Variance covariance matrix - ATP and COVAR versions**

In Excel 2007 and earlier, there is only one covariance function, **COVAR** and it returns the population covariance for two return vectors. In figure 5, rows 39 to 43, the **COVAR** function uses Excel range names for each of the return vectors. The return values are population estimates.

Construction of the individual cell formulas can be simplified by using range names with the **INDIRECT** function. To do this:

- Copy and paste the stock codes vector to the range G46:J46.
- Using **Paste Special > Transpose**, paste the transposed stock codes vector to F47.

- Enter the formula =COVAR(INDIRECT(F\$47),INDIRECT(\$G46)) at G47.
- Copy and paste the formula to complete the variance covariance matrix.

The return values, and cell formulae are shown in figure 6.

	A	B	C	D	E	F	G	H	I	J	K	L	M
45													
46								AGK	CSL	SPN	SKT		
47						AGK	0.001764	-0.000689	0.000338	0.000608			
48						CSL	-0.000689	0.003009	0.000127	-0.000599			
49						SPN	0.000338	0.000127	0.001317	0.000223			
50						SKT	0.000608	-0.000599	0.000223	0.001411			
51													
52							G47: =COVAR(INDIRECT(\$F47),INDIRECT(G\$46))						
53							G48: =COVAR(INDIRECT(\$F48),INDIRECT(G\$46))						
54							G49: =COVAR(INDIRECT(\$F49),INDIRECT(G\$46))						
55							G50: =COVAR(INDIRECT(\$F50),INDIRECT(G\$46))						
56													
57								H47: =COVAR(INDIRECT(\$F47),INDIRECT(H\$46))					
58								H48: =COVAR(INDIRECT(\$F48),INDIRECT(H\$46))					
59								H49: =COVAR(INDIRECT(\$F49),INDIRECT(H\$46))					
60								H50: =COVAR(INDIRECT(\$F50),INDIRECT(H\$46))					
61													
62													

Fig 6: Variance covariance matrix - COVAR and INDIRECT version

### 3. Covariance with VBA

The Excel 2007 **COVAR** function returns the population covariance. To estimate the sample covariance, the custom function Covar\_s has been developed.

Here is the code.

#### VBA: Covar\_s (Available in the XLFProject.XLF\_Module)

```

Function Covar_s(InArray1 As Variant, InArray2 As Variant) As Variant

Dim NumRows As Long, NumRows1 As Long, NoRows2 As Long
Dim i As Long, j As Long
Dim InArrayType1 As String
Dim InArrayType2 As String
Dim InA1Ave As Double, InA2Ave As Double

Dim Temp1() As Double, Temp2 As Double

10 On Error GoTo ErrHandler

20 InArrayType1 = TypeName(InArray1)
30 InArrayType2 = TypeName(InArray2)

40 If InArrayType1 = "Range" Then
50     NumRows = UBound(InArray1.Value2, 1) - _
        LBound(InArray1.Value2, 1) + 1
60 ElseIf InArrayType1 = "Variant()" Then
70     NumRows = UBound(InArray1, 1) - LBound(InArray1, 1) + 1
80 Else
90     GoTo ErrHandler
100 End If

```

```

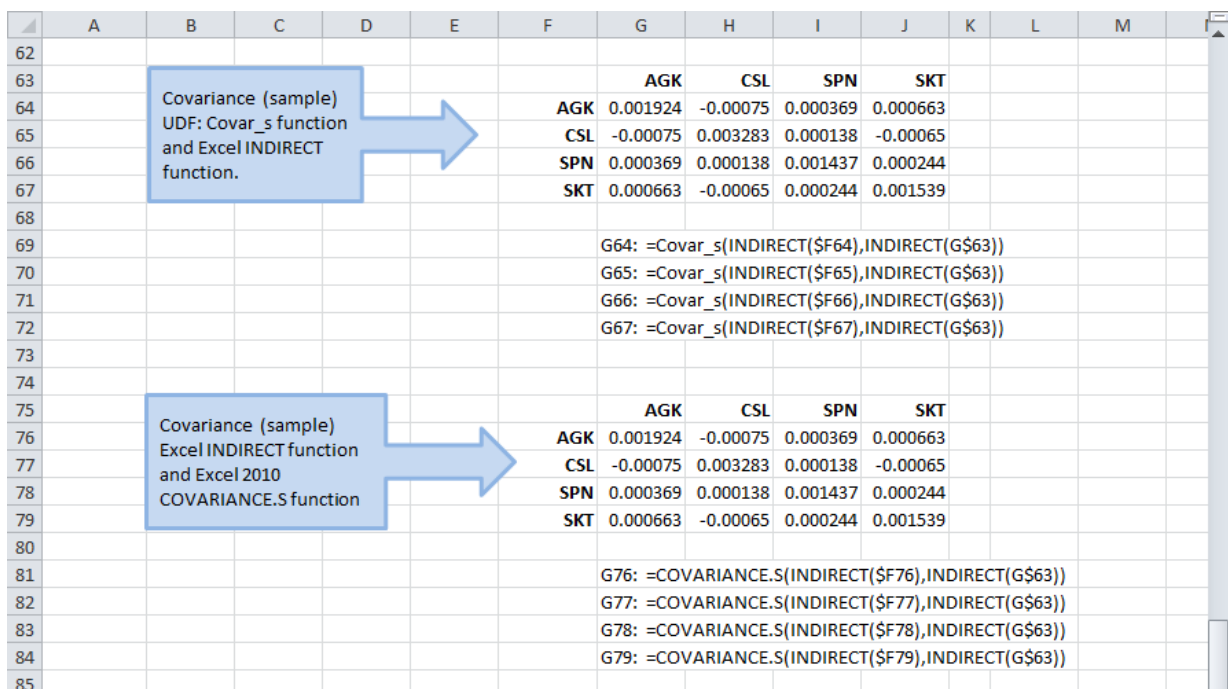
110 ReDim Temp1(1 To NumRows)
120 With Application.WorksheetFunction
130 InA1Ave = .Average(InArray1)
140 InA2Ave = .Average(InArray2)
150     For i = 1 To NumRows
160         Temp1(i) = (InArray1(i) - InA1Ave) * (InArray2(i) - InA2Ave)
170     Next i
180     Temp2 = .Sum(Temp1) / (NumRows - 1)
190 End With
200     Covar_s = Temp2
210 Exit Function
ErrorHandler:
220     Covar_s = CVErr(xlErrNA)
End Function

```

**Covar\_s** is the sample version of the  $\sigma_{ij}$  equation with  $n - 1$  in the denominator rather than  $n$ . The sample covariance is calculated in lines 150 to 180 of the code. The `For ... Next` loop at lines 150 to 170 returns the numerator of the equation as a vector to the array named `Temp1`. The sum of `Temp1` `.Sum(Temp1)` is then divided by  $n - 1$  in line 180.

**Covar\_s** is useful if you open the workbook on an Excel 2007 or earlier platform. If you only use Excel 2010 or later, then **COVARIANCE.S** or **COVARIANCE.P** are available.

Figure 7 show the output for the **Covar\_s** custom function, and the Excel 2010 **COVARIANCE.S** function.



**Fig 7: UDF Covar\_s and Excel 2010 COVARIANCE.S**

The example in figure 7 uses a nested function in combination with range names.

The next code section provides a custom function to return the variance-covariance as a single array formula. It uses the Excel 2007 **COVAR** function, for the population covariance, at line 170, but the code is easily modified. The function is named **VarCovar\_p** and is entered as an array CSE formula.

### VBA: VarCovar\_p (Available in the XLFProject.XLF\_Module)

```

Function VarCovar_p(InMatrix As Variant) As Variant

Dim NumRows As Long, numCols As Long
Dim i As Long, j As Long
Dim InMatrixType As String
Dim Temp() As Double

10 On Error GoTo ErrHandler

20 InMatrixType = TypeName(InMatrix)

30 If InMatrixType = "Range" Then
40     NumRows = UBound(InMatrix.Value2, 1) - _
        LBound(InMatrix.Value2, 1) + 1
50     numCols = UBound(InMatrix.Value2, 2) - _
        LBound(InMatrix.Value2, 2) + 1
60     ReDim Temp(1 To numCols, 1 To numCols)
70 ElseIf InMatrixType = "Variant()" Then
80     NumRows = UBound(InMatrix, 1) - LBound(InMatrix, 1) + 1
90     numCols = UBound(InMatrix, 2) - LBound(InMatrix, 2) + 1
100    ReDim Temp(1 To numCols, 1 To numCols)
110 Else
120     GoTo ErrHandler

130 End If

140 With Application.WorksheetFunction
150     For i = 1 To numCols
160         For j = 1 To numCols
170             Temp(i, j) = .Covar(.Index(InMatrix, 0, i), _
                .Index(InMatrix, 0, j))
180         Next j
190     Next i
200 End With

210     VarCovar_p = Temp
220 Exit Function
ErrorHandler:
230     VarCovar_p = CVErr(xlErrNA)

End Function

```

To use **VarCovar\_p** function do the following:

- Determine the dimensions of the returned variance-covariance (VCV) matrix.
- Give the returns data at G9:J21 a name, such as Returns.
- Select the range where the result is to be returned to.
- In the formula bar enter =VarCovar\_p>Returns)
- Hit Control+Shift+Enter to complete the array formula.
- Add labels as shown in figure 5.

	A	B	C	D	E	F	G	H	I	J	K	L	M
38													
39							AGK	CSL	SPN	SKT			
40		Covariance using Excel 2007 COVAR function with range names.				AGK	0.001764	-0.000689	0.000338	0.000608		J40: =COVAR(AGK,SKT)	
41	CSL					-0.000689	0.003009	0.000127	-0.000599	J41: =COVAR(CSL,SKT)			
42	SPN					0.000338	0.000127	0.001317	0.000223	J42: =COVAR(SPN,SKT)			
43	SKT					0.000608	-0.000599	0.000223	0.001411	J43: =COVAR(SKT,SKT)			
86													
87							AGK	CSL	SPN	SKT			
88		Covariance (population) UDF: Array formula VarCovar_p as CSE.				AGK	0.001764	-0.00069	0.000338	0.000608			
89	CSL					-0.00069	0.003009	0.000127	-0.0006				
90	SPN					0.000338	0.000127	0.001317	0.000223				
91	SKT					0.000608	-0.0006	0.000223	0.001411				
92													
93						G88: =VarCovar_p>Returns)							
94						G89: =VarCovar_p>Returns)							
95						G90: =VarCovar_p>Returns)							
96						G91: =VarCovar_p>Returns)							
97													

Fig 8: User defined array function - VarCovar\_p

In the next section we examine the correlation coefficient.

### 4. Correlation

Correlation is a standardized measure of co-movement. The formula to calculate the correlation  $\rho_{i,j}$  between the returns for stocks  $i$  and  $j$  is

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$

where  $\sigma_{i,j}$  is the covariance, and  $\sigma_i$  and  $\sigma_j$  are the standard deviation, and stocks  $i$  and  $j$  respectively. The Excel function is **CORREL**.

#### Excel

CORREL(array1,array2)	Returns the correlation coefficient between two data sets
-----------------------	---

The correlation coefficient  $\rho$  is bounded in the range  $-1 \leq \rho_{i,j} \leq 1$ . A correlation of -1 is perfect negative correlation, a correlation of +1 is perfect positive correlation, and a correlation of 0 represents zero correlation.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
97														
98							AGK	CSL	SPN	SKT				
99		Correlation: Excel CORREL function and INDIRECT function.				AGK	1.00000	-0.29900	0.22179	0.38544				
100	CSL					-0.29900	1.00000	0.06374	-0.29078					
101	SPN					0.22179	0.06374	1.00000	0.16381					
102	SKT					0.38544	-0.29078	0.16381	1.00000					
103														
104						G99: =CORREL(INDIRECT(\$F99),INDIRECT(G\$98))								
105						G100: =CORREL(INDIRECT(\$F100),INDIRECT(G\$98))								
106						G101: =CORREL(INDIRECT(\$F101),INDIRECT(G\$98))								
107						G102: =CORREL(INDIRECT(\$F102),INDIRECT(G\$98))								
108														

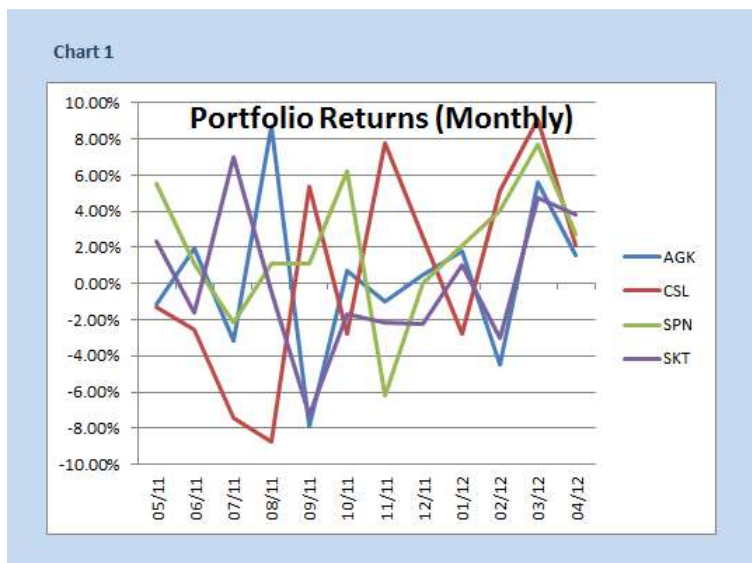
Fig 9: Correlation matrix - for the four stock portfolio

The correlation coefficient is invariant to the use of population or sample estimates. Provided the numerator and denominator are all population, or all sample estimates, the returned value is the same. In other words, apply the consistency principle. See figure 10 for returned values, in both cases the value is -0.2990.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
108														
109		<b>Validation</b>												
110		Population												
111			Covar <sub>P</sub> <sub>CSL,AGK</sub>		-0.00069		E109: -0.000688833091647805							
112			SD <sub>P</sub> <sub>CSL</sub>		0.054857		E110: 0.0548569193365494							
113			SD <sub>P</sub> <sub>AGK</sub>		0.041997		E111: 0.041996679555981							
114			Corr <sub>CSL,AGK</sub>		-0.29900		E112: =E109/(E110*E111)							
115														
116		Sample												
117			Covar <sub>S</sub> <sub>CSL,AGK</sub>		-0.00075		E114: -0.000751454281797605							
118			SD <sub>S</sub> <sub>CSL</sub>		0.057296		E115: 0.0572961835863439							
119			SD <sub>S</sub> <sub>AGK</sub>		0.043864		E116: 0.0438641012101665							
120			Corr <sub>CSL,AGK</sub>		-0.29900		E117: =E114/(E115*E116)							
121														

**Fig 10: Correlation coefficient** - from population and sample estimates

Excel has limited ability to produce a 3D scatter plot of the correlation matrix in figure 9 and a line chart line charts of the return series (figure 11), can be difficult to interpret.



**Fig 11: Plot of monthly returns**

Instead we produce pair wise scatter plots of selected correlation relationships.

From the data in figure 9, the key features for the  $-1 \leq \rho_{i,j} \leq 1$  are:

- Largest positive correlation: AGK, SKT: +0.385 (Figure 12)
- Largest negative correlation: AGK, CSL: -0.299 (Figure 13)
- Smallest correlation: SPN, CSL: 0.064 (Figure 14)



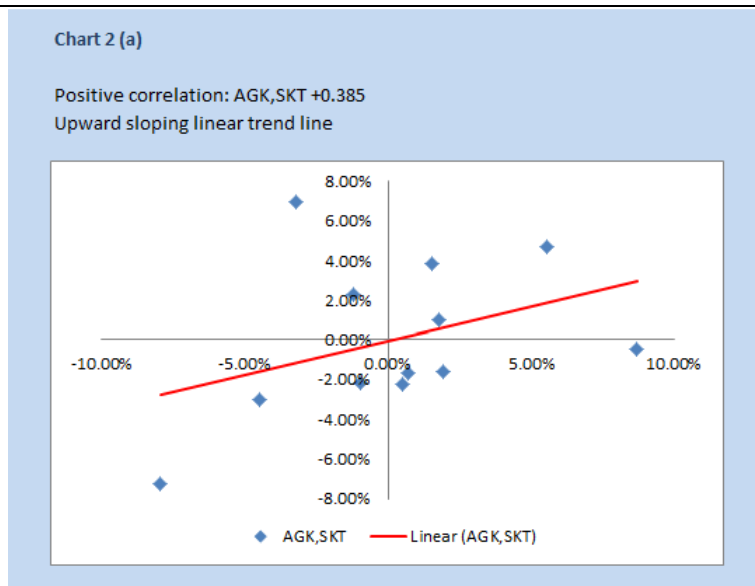


Figure 12: Largest positive correlation: AGK, SKT: +0.385

The positive correlation for AGK, SKT is shown by the positive (upward) slope of linear trend line in figure 12. In contrast, the negative correlation between the returns for AGK and CSL is shown by the negative (downward) slope of the linear trend line (figure 13)

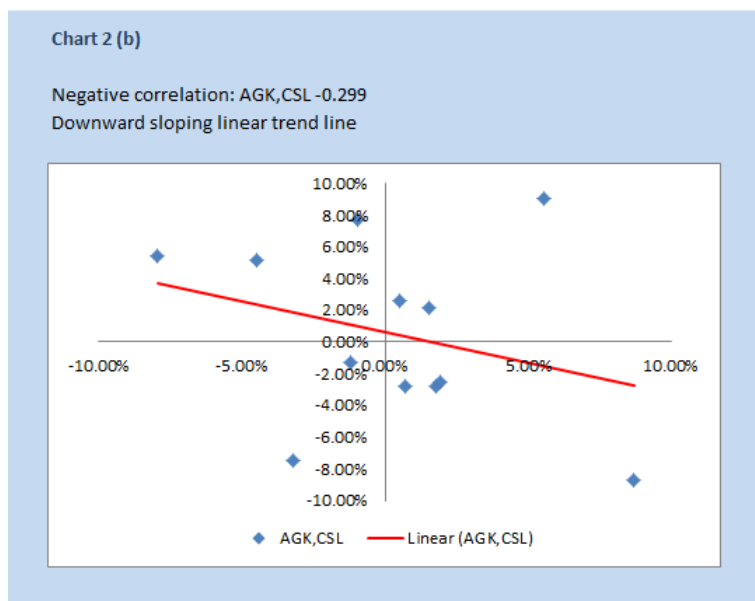


Fig 13: Largest negative correlation - AGK, CSL: -0.299

In the case where the correlation is close to zero, then the linear trend line is more flat, as shown in figure 14.

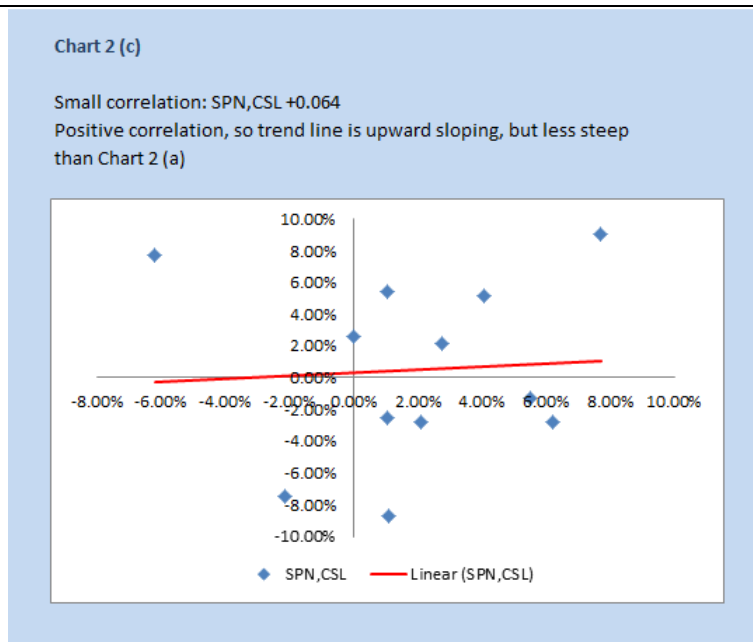


Figure 14: Smallest correlation: SPN, CSL: 0.064

## 5. The portfolio

The returns  $r_p$  on a portfolio are the weight sum of the returns for the individual assets

$$r_p = \sum_{i=1}^N \omega_i r_i$$

If the investor has portfolio weights of  $a = 0.1, b = 0.2, c = 0.4, d = 0.3$ , then the portfolio return is shown in figure 15.

	A	B	C	D	E	F	G	H	I	J	K	L	M
4													
5													
6		Portfolio returns.		AGK	CSL	SPN	SKT						
7		Average		0.002484	0.005424	0.019349	0.000391						
8		Weights		0.1	0.2	0.4	0.3						
9													
10		$r_p =$		0.00919					C9: =SUM(D6*D7,E6*E7,F6*F7,G6*G7)				
11		$r_p =$		0.00919					C11: =SUM(Weights*Average)				
12		$r_p =$		0.00919					C12: =SUMPRODUCT(Average,Weights)				
13		$r_p =$		0.00919					C13: =MMULT(Weights,TRANSPOSE(Average))				
14													
15													

Fig 15: Portfolio returns

In figure 15, portfolio returns are estimated using the formula, line 10.

The Excel **SUMPRODUCT** function is in row 12, and array formulae, are in rows 11 and 13. Both require **Control+Shift+Enter**.

The variance of the returns  $\sigma_p^2$  on a portfolio are estimated by the double summation formula

$$\sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \sigma_{i,j}$$

For a four asset portfolio, the variance of returns is:

$$\sigma_p^2 = \omega_a^2 \sigma_a^2 + \omega_b^2 \sigma_b^2 + \omega_c^2 \sigma_c^2 + \omega_d^2 \sigma_d^2 + 2\omega_a \omega_b \sigma_{a,b} + 2\omega_a \omega_c \sigma_{a,c} + 2\omega_a \omega_d \sigma_{a,d} + 2\omega_b \omega_c \sigma_{b,c} + 2\omega_b \omega_d \sigma_{b,d} + 2\omega_c \omega_d \sigma_{c,d}$$

where the stocks are indexed  $\{a, b, c, d\}$ . See rows 23 to 33 of the worksheet in figure 16.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
15															
16		Portfolio variance													
17		Population covariance				AGK	CSL	SPN	SKT						
18		AGK	0.001764	-0.00069	0.000338	0.000608									
19		CSL	-0.00069	0.003009	0.000127	-0.0006									
20		SPN	0.000338	0.000127	0.001317	0.000223									
21		SKT	0.000608	-0.0006	0.000223	0.001411									
22															
23			1.76372E-05						C23: =D8^2*D18						
24			0.000120371						C24: =E8^2*E19						
25			0.000210798						C25: =F8^2*F20						
26			0.000126995						C26: =G8^2*G21						
27			-2.75533E-05						C27: =2*D8*E8*E18						
28			2.7047E-05						C28: =2*D8*F8*F18						
29			3.6483E-05						C29: =2*D8*G8*G18						
30			2.03077E-05						C30: =2*E8*F8*F19						
31			-7.19029E-05						C31: =2*E8*G8*G19						
32			5.36045E-05						C32: =2*F8*G8*G20						
33		Var <sub>p</sub>	0.000513787						C33: =SUM(C23:C32)						
34		Var <sub>p</sub>	0.000513787						C34: =MMULT(Weights,MMULT(VCV,TRANSPOSE(Weights)))						
35															
36		SD <sub>p</sub>	0.02266688						C36: =SQRT(C34) 2.26668804892243E-02						
37		SD <sub>p</sub>	2.27%						C37: =C36 2.26668804892243E-02						
38															

**Fig 16: Portfolio variance** - calculated using the algebraic version (rows 23 to 33), and the shorter matrix version (row 34)

We can also use the Excel array formula to estimate the variance

**Excel**

MMULT(array1,array2)	Returns the matrix product of two arrays
TRANSPOSE(array)	Returns the transpose of an array

In matrix notation, the portfolio variance is

$$\sigma_p^2 = \omega \Sigma \omega^T$$

Where  $\omega$  is a row vector of weights,  $\omega^T$  is its transpose, and  $\Sigma$  is the variance-covariance matrix. In Excel, using the range name **Weights** for  $\omega$  and **VCV** for  $\Sigma$ , the cell formula is

=MMULT(Weights,MMULT(VCV,TRANSPOSE(Weights)))

entered as an array formula. See row 34 of the worksheet in figures 16 and 17.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
4															
5															
6		Portfolio returns.		AGK	CSL	SPN	SKT								
7		Average		0.002484	0.005424	0.019349	0.000391								
8		Weights		0.1	0.2	0.4	0.3								
9															
14		$r_p =$		0.5											
15															
16		Portfolio variance													
17		Population covariance		AGK	CSL	SPN	SKT								
18		AGK		0.001764	-0.00069	0.000338	0.000608								
19		CSL		-0.00069	0.003009	0.000127	-0.0006								
20		SPN		0.000338	0.000127	0.001317	0.000223								
21		SKT		0.000608	-0.0006	0.000223	0.001411								
22															
23				1.76372E-05											
24				0.000120371											
25				0.000210798											
26				0.000126995											
27				-2.75533E-05											
28				2.7047E-05											
29				3.6483E-05											
30				2.03077E-05											
31				-7.19029E-05											
32				5.36045E-05											
33		Var <sub>p</sub>		0.000513787											
34		Var <sub>p</sub>		0.000513787											
35															
36		SD <sub>p</sub>		0.02266688											
37		SD <sub>p</sub>		2.27%											
38															

C18	=SUM(D7:D8,E7*E8,F7*F8)
C23	=D8^2*D18
C24	=E8^2*E19
C25	=F8^2*F20
C26	=G8^2*G21
C27	=2*D8*E8*E18
C28	=2*D8*F8*F18
C29	=2*D8*G8*G18
C30	=2*E8*F8*F19
C31	=2*E8*G8*G19
C32	=2*F8*G8*G20
C33	=SUM(C23:C32)
C34	=MMULT(Weights,MMULT(VCV,TRANSPOSE(Weights)))
C36	=SQRT(C34) 2.26668804892243E-02
C37	=C36 2.26668804892243E-02

Fig 17: Portfolio variance in Excel function format.

## 6. Portfolio charts

The x-y scatter plot of the portfolio and components is shown in figure 18. The figure indicates that the return for SPN is almost four times greater than the other components. Due to the covariance structure, the standard deviation of portfolio is lower than the standard deviation of any of the component stocks.

In the next section we describe a method to rebalance the portfolio, in the case where the investor wants to achieve a different risk return tradeoff.

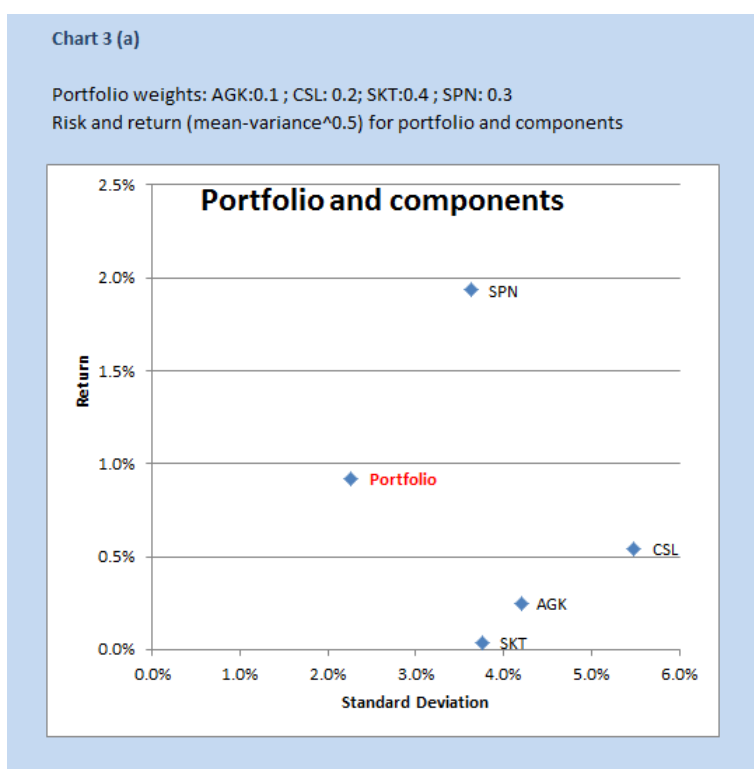


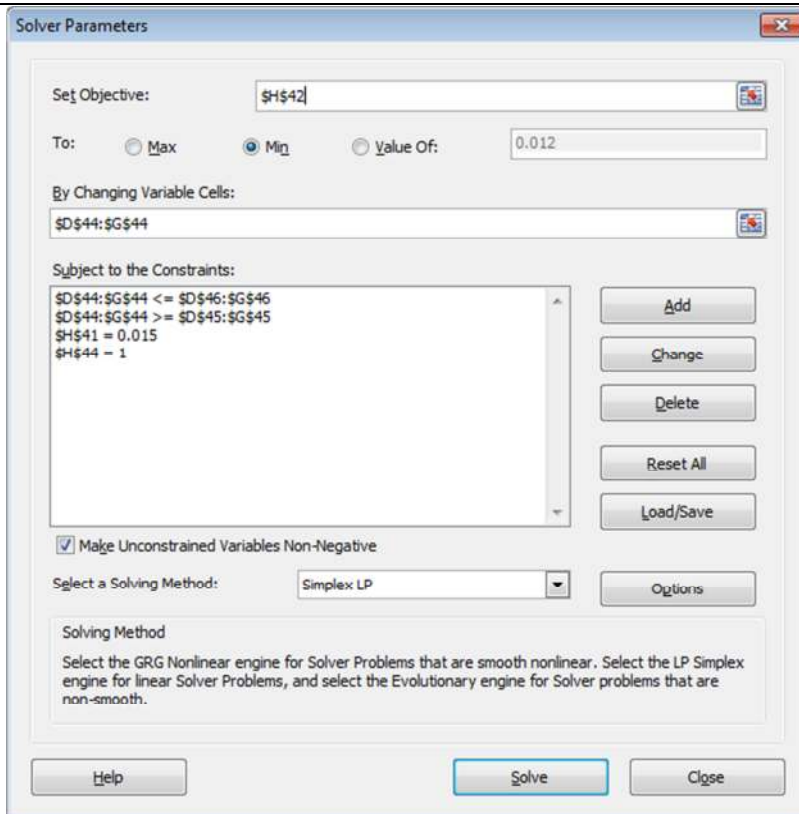
Fig 18: Portfolio and components

## 7. Portfolio optimization

**Target 1:** Suppose that the investor wants to rebalance the portfolio to achieve a target return of 1.5% per month with the lowest possible standard deviation. The investor assumes that the historical information can be used as an estimate of expected returns and risk in the future. In addition bounds are set on the proportion of funds in each stock. The weights in the individual stocks AGK, CSL, and SPN must be in the range 10% to 30%, the weight in SKT must not be less than 10% and must not exceed 60%. By definition, the weights must sum to 1.

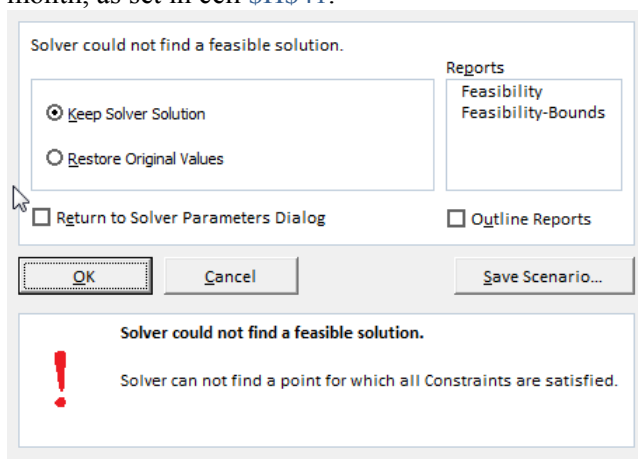
To solve the portfolio mix, subject to the constraints imposed, we will use the SOLVER add-in.

The **SOLVER** add-in is located on the Data tab, and the Parameter settings are shown in figure 19. We use the Set Objective, to Minimize the standard deviation of the portfolio, cell  $\$H\$42$ . The weights in range  $\$D\$44:\$G\$44$ , are set in the By Changing Variable Cells parameter, and the constraints are set in the Subject to the Constraints parameter.



**Fig 19: Setting the Solver Parameters - for Target 1**

When the scenario for Target 1 is run, we find that Solver could not find a feasible solution. In other words, subject to the constraints, no combination of stocks achieved the target return of 1.5% per month, as set in cell [\\$H\\$41](#).



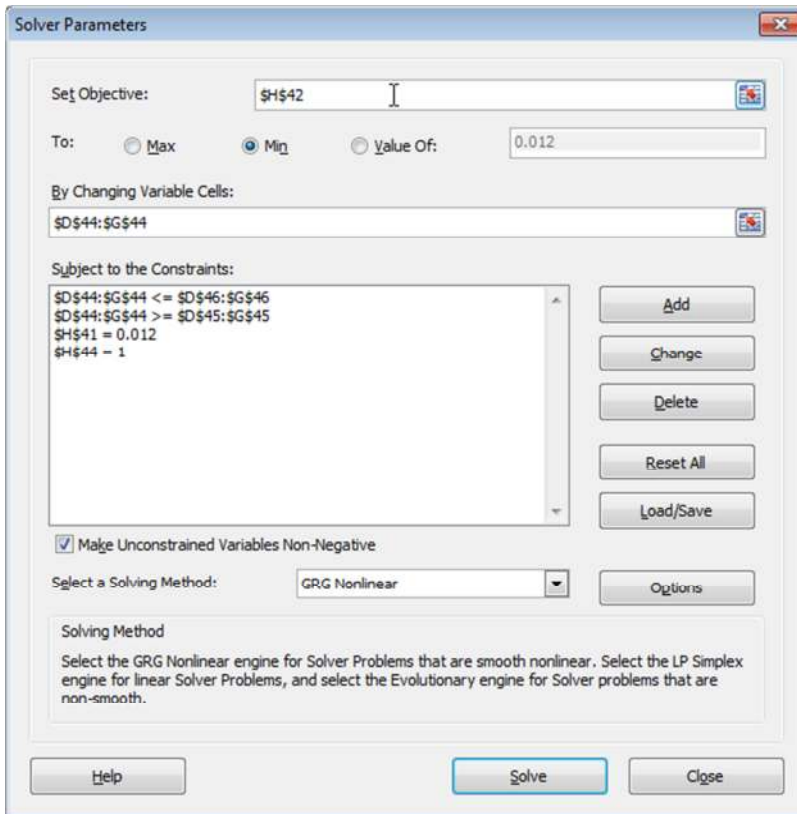
**Fig 20: No feasible solution - with Target 1 constraints, using the Simplex LP method.**

After some experimentation, the set of constraints in Target 2 is evaluated.

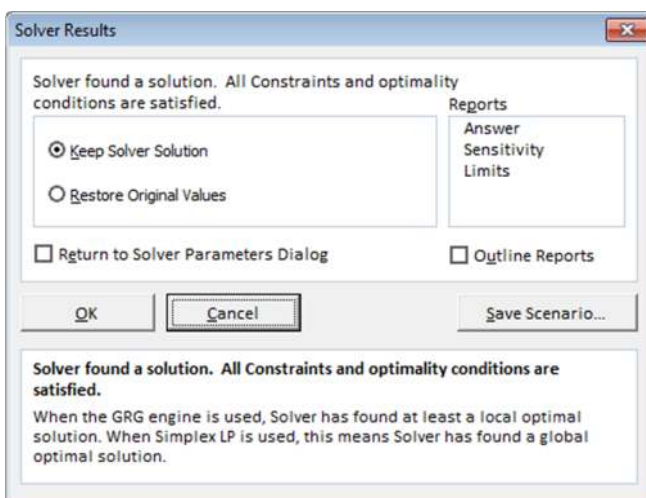
**Target 2:** In this scenario, the maximum weight for SPN is increased to 60%, and the target return is set to 1.2% per month. See figure 21, for the weights range, and figure 22 for the revised Solver Parameter set.

	A	B	C	D	E	F	G	H	I	J	K	L
40		Portfolio returns.		AGK	CSL	SPN	SKT	Port				
43												
45		Constraints	MinW	0.10	0.10	0.10	0.10					
46			MaxW	0.30	0.30	0.60	0.60					
47												

**Fig 21: Target 2** - revised weight constraints. SPN has increased to 0.6 weight.



**Fig 22: Target 2** - with revised constraints, and also selecting the GRG Nonlinear method. After switching to the GRG Nonlinear method, a solution was found as shown in figure 23.



**Fig 23: Solution found.**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
40		Portfolio returns.		AGK	CSL	SPN	SKT	Port							
41		Average		0.2484%	0.5424%	1.9349%	0.0391%	1.2000%	H41: =SUMPRODUCT(D41:G41,D44:G44)						
42		Std Dev.p		4.1997%	5.4857%	3.6297%	3.7564%	2.4918%	H42: =SQRT(MMULT(D44:G44,MMULT(VCV,TRANSPOSE(D44:G44))))						
43															
44		W		0.12	0.18	0.55	0.15	1.00	H44: =SUM(D44:G44)						
45		Constraints	MinW	0.10	0.10	0.10	0.10								
46			MaxW	0.30	0.30	0.60	0.60								
47															

Fig 24: The solution weight

## 8. Portfolio variance with VBA

In this section, is the VBA code for the function PortSD to estimate the portfolio standard deviation.

### VBA: PortSD (Available in the XLFProject.XLF\_Module)

```

Function PortSD(Weights As Range, VCV As Range) As Double
    Dim PortVar As Variant
10  With Application.WorksheetFunction
20      PortVar = .MMult(Weights, .MMult(VCV, .Transpose(Weights)))
30      PortSD = Sqr(.Sum(PortVar))
40  End With
End Function

```

The weights range is a row vector. The function uses the Excel functions MMULT, TRANSPOSE, and SUM. In row 30, is the VBA square root function, SQR.

The procedure has two important aspects. In line 20, the portfolio variance is calculated. The return value from the formula is a 1 x 1 array, thus PortVar is of the type Variant. VBA cannot take the square root of a 1 x 1 array, so in line 30, we take the sum of the array, to return a non-array number, then take the square root.

#### Resources

- The Excel file for this is available at: <http://excelatfinance.com/xlf/xlf-portfolio-analysis-v2.xlsm>
- An online version of section 1 Descriptive Statistics, and section 2 covariance is available at: <http://excelatfinance.com/online/topic/portfolio-analysis-excel/>

#### Portfolio Analysis

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Author: Ian O'Connor :: ioconnor@excelatfinance.com