THE MERTON MODEL OF RISKY DEBT

Merton (1974) and Black and Scholes (1973) proposed a model to link the credit risk of a firm to its capital structure. The capital structure comprises a zero coupon bond, equity with no dividend payments, and the firm's asset value is assumed to follow a lognormal diffusion process.

Equity value and the probability of default on debt payments

Following Hull, Nelken and White (2004). Let A_0 be the firm's asset value, B_0 be the value of debt, assumed to be a zero coupon bond paying B at maturity, and E_0 be the equity value, all at time 0.

If at time τ, the asset value A exceeds the bond value, then the bond holders are paid out and the equity holders receive the residual payment. In the case where the asset value is less than the bond value, then the firm defaults and the bond holders receive the available asset value and the equity holders receive nothing.

Thus, in the Merton framework, the value of equity at maturity, time τ, is given by

$$E_\tau = Max[A_\tau - B, 0]$$

Equity is equivalent to a call option on the assets of the firm with an exercise price equal to the face value of the bond. Using the formula for the price of a call option, the value of equity E_0 at time 0 is

$$E_0 = A_0[N(d_1) - L \cdot N(d_2)] \tag{1}$$

where $L = Be^{-i\tau}/A_0$ is the leverage ratio, and

$$d_1 = -\frac{log(L)}{\sigma_A\sqrt{\tau}} + 0.5\sigma_A\sqrt{\tau}; \quad d_2 = d_1 - \sigma_A\sqrt{\tau} \tag{2}$$

i is the risk free interest rate, σ_A is the volatility of asset value, both of which are assumed to be constant.

From equation 1, the equity value E_0 is a function of the asset value A_0. Jones, Mason and Rosenfeld (1984) have shown that Itô's lemma can be used to determine the instantaneous volatility of the firm's equity σ_E

$$E_0\sigma_E = \frac{\partial E}{\partial A}A_0\sigma_A$$

giving

$$\sigma_E = \frac{\sigma_A N(d_1)}{N(d_1) - L \cdot N(d_2)} \tag{3}$$

The risk-neutral probability P, that the equity holders will not exercise their call option to buy the assets for price B at time τ is given by

$$P = N(-d_2) \tag{4}$$

and P is a function of asset volatility σ_A, leverage L, and time to repayment τ. This is equivalent to the probability of default.

Debt valuation and implied credit spreads

The market value of debt at time 0 is given by

$$B_0 = A_0 - E_0$$

and from equation 1 B_0becomes

$$B_0 = A_0\big(N(-d_1) + L \cdot N(d_2)\big) \tag{5}$$

By implication, the yield to maturity, r , the rate on the risky debt, is defined as

$$B_0 = De^{-i\tau} = D^* e^{(r-i)\tau} \tag{6}$$

Substituting equation 6 into equation 5 gives the yield to maturity as

$$r = i - ln\frac{(N(d_2) + N(-d_1)/L)}{\tau}$$

and s is the credit spread implied by the Merton model

$$s = r - i = -ln\frac{(N(d_2) + N(-d_1)/L)}{\tau} \tag{7}$$

Applying the HNW (2004) specification to the S & A (2002) example

Saunders and Allan (2002) illustrate the Merton model with the following sample data

B = \$100,000	
τ = 1 year	
I = 5% pa	The risk free interest rate of the same maturity
L = 0.9	The leverage ratio
σ_A=12%	The volatility of the assets of the firm

Equation 2:

$$d_1 = -\left(\frac{\log(0.9)}{0.12 \times \sqrt{1}}\right) + 0.5 \times \left(0.12 \times \sqrt{1}\right) = 0.938004$$

$$d_2 = 0.938004 - \left(0.12 \times \sqrt{1}\right) = 0.081804$$

$$N(d_1) = 0.825879$$

$$N(-d_1) = 0.174121$$

$$N(d_2) = 0.793323$$

Equation 5:

$$B_0 = \frac{\$100{,}000e^{-0.05\times 1}}{0.9} \times (0.174121 + 0.9 \times 0.793323) \approx \$93{,}866.42$$

Equation 7:

$$s = -log\frac{0.793323 + \frac{0.174121}{0.9}}{1} = 0.0132975 \approx 1.3297\%$$

Reconciling to the S & A (2002) specification

Saunders and Allan (2002) use different expression in the formulation of the model.

From equation 2 Saunders and Allan:

$$h_1 = -[0.5\sigma^2\tau - log(L)]/\sigma\sqrt{\tau}$$

$$h_2 = -[0.5\sigma^2\tau + log(L)]/\sigma\sqrt{\tau}$$

$$h_1 = -\frac{[0.5 \times 0.12^2 \times 1 - log(0.9)]}{0.12 \times \sqrt{1}} = -0.938004$$

$$h_2 = -\frac{[0.5 \times 0.12^2 \times 1 + log(0.9)]}{0.12 \times \sqrt{1}} = 0.818004$$

$$N(h_1) = 0.174121$$

$$N(h_2) = 0.793323$$

The debt price from Equation 5:

$$B_0 = Be^{-i\tau}\big(N(h_2) + (1/L) \times N(h_1)\big)$$

$$B_0 = \$100{,}000e^{-0.05\times 1}(0.793323 + (1/0.9) \times 0.174121) \approx \$93{,}866.42$$

The implied credit spread from Equation 7:

$$s = \frac{-1}{\tau}log\big(N(h_2) + (1/L) \times N(h_1)\big)$$

$$s = \frac{-1}{1}log(0.793323 + (1/0.9) \times 0.174121) = 0.013297 \approx 1.3297\%$$

References

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